# Capítulo 19

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1. **Dada la acción siguiente:**

$$S = \frac{1}{2}\int d^4x[\delta_\mu\phi\delta^\mu\phi - m^2\phi^2]$$

a) **Demostrar que L =** $\frac{1}{2}\left[\delta_\mu\phi\delta^\mu\phi - m^2\phi^2\right]$ **es invariante bajo la transformación:**

$$x^{0'} = \gamma x^0 - \gamma\beta x^1$$
$$x^{1'} = -\gamma\beta x^0 + \gamma x^1$$
$$x^{2'} = x^2$$
$$x^{3'} = x^3$$

b) **Calcular** $\frac{\delta S}{\delta\phi}$

*a)* $\partial_0\phi = \partial^0\phi = \frac{\delta\phi}{\delta x^{0'}}\cdot\frac{\delta x^{0'}}{\delta x^0} + \frac{\delta\phi}{\delta x^{1'}}\cdot\frac{\delta x^{1'}}{\delta x^0} + \frac{\delta\phi}{\delta x^{2'}}\cdot\frac{\delta x^{2'}}{\delta x^0} + \frac{\delta\phi}{\delta x^{3'}}\cdot\frac{\delta x^{3'}}{\delta x^0} =$

$= \delta_{0'}\phi\cdot\gamma - \delta_{1'}\phi\cdot\gamma\beta = \gamma(\delta_{0'} - \beta\delta_{1'})\phi$

$\partial_0\phi\partial^0\phi = \gamma\left(\delta_{0'} - \beta\delta_{1'}\right)\gamma\left(\delta_{0'} - \beta\delta_{1'}\right)\phi = \gamma^2(\delta_{0'}^2 + \beta^2\delta_{1'}^2 - 2\beta\delta_{0'}\delta_{1'})\phi$

$\partial_1\phi = -\partial^1\phi = \frac{\delta\phi}{\delta x^{0'}}\cdot\frac{\delta x^{0'}}{\delta x^1} + \frac{\delta\phi}{\delta x^{1'}}\cdot\frac{\delta x^{1'}}{\delta x^1} + \frac{\delta\phi}{\delta x^{2'}}\cdot\frac{\delta x^{2'}}{\delta x^1} + \frac{\delta\phi}{\delta x^{3'}}\cdot\frac{\delta x^{3'}}{\delta x^1} =$

$= -\delta_{0'}\phi\cdot\gamma\beta + \delta_{1'}\phi\cdot\gamma = \gamma(-\beta\delta_{0'} + \delta_{1'})\phi$

$\partial_1\phi\partial^1\phi = \gamma\left(-\beta\delta_{0'} + \delta_{1'}\right)\gamma\left(\beta\delta_{0'} - \delta_{1'}\right)\phi = \gamma^2\left(-\beta^2\delta_{0'}^2 - \delta_{1'}^2 + 2\beta\delta_{0'}\delta_{1'}\right)\phi$

$\partial_2\phi\partial^2\phi = -\delta_{2'}^2\phi$

$\partial_3\phi\partial^3\phi = -\delta_{3'}^2\phi$

*Sabemos que* $\gamma^2 = \frac{1}{1-\beta^2} \Rightarrow \gamma^2\left(1-\beta^2\right) = 1$

$L = \frac{1}{2}\left[\delta_\mu\phi\delta^\mu\phi - m^2\phi^2\right] = \frac{1}{2}[\gamma^2\left(\delta_{0'}^2 + \beta^2\delta_{1'}^2 - 2\beta\delta_{0'}\delta_{1'}\right)\phi + \gamma^2\left(-\beta^2\delta_{0'}^2 - \delta_{1'}^2 + 2\beta\delta_{0'}\delta_{1'}\right)\phi +$
$-\delta_{2'}^2\phi - \delta_{3'}^2\phi - m^2\phi^2] = \frac{1}{2}[\gamma^2(1-\beta^2)\delta_{0'}^2\phi + \gamma^2(\beta^2-1)\delta_{1'}^2\phi - \delta_{2'}^2\phi - \delta_{3'}^2\phi - m^2\phi^2] =$
$= \frac{1}{2}\left[\delta_{0'}^2\phi - \delta_{1'}^2\phi - \delta_{2'}^2\phi - \delta_{3'}^2\phi - m^2\phi^2\right] = \frac{1}{2}\left[\delta_{\mu'}\phi\delta^{\mu'}\phi - m^2\phi^2\right]$

Lo que demuestra que ante una transformación Lorentz "L" es invariante.

b) $\mathrm{L} = \frac{1}{2}\left[\delta_\mu \phi \delta^\mu \phi - m^2\phi^2\right] = \frac{1}{2}\left[\delta_0^2\phi - \delta_1^2\phi - \delta_2^2\phi - \delta_3^2\phi - m^2\phi^2\right]$

$$\frac{\delta S}{\delta \phi} = \frac{\delta L}{\delta \phi} - \delta_\mu \left(\frac{\delta L}{\delta\left(\delta_\mu \phi\right)}\right) = -m^2\phi - \left(\delta_0^2\phi - \delta_1^2\phi - \delta_2^2\phi - \delta_3^2\phi\right) =$$

$$= -\delta_0^2\phi + \delta_1^2\phi + \delta_2^2\phi + \delta_3^2\phi - m^2\phi$$